Response Modeling – Marketing Engineering Technical Note [1]

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Introduction

In this note, we first describe the simplest of the response model types: aggregate response to a single marketing instrument in a static, noncompetitive environment. Then we introduce additional marketing instruments, dynamics, and competition into the aggregate response model. The interested reader is referred to Lilien, Kotler and Moorthy (1992) and Leeflang et al (2000) for additional details.

Before we proceed, we need to be clear on the vocabulary:

We use several terms to denote the equation or sets of equations that relate

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[1] This technical note is a supplement to the materials in Chapter 1 of Principles of Marketing Engineering, by Gary L. Lilien, Arvind Rangaswamy, and Arnaud De Bruyn (2007). Visit www.decisionpro.biz for additional information.

dependent variables to *independent variables* in a model, such as *relationship*, *specification*, and *mathematical form*.

Parameters are the constants (usually the a's and b's) in the mathematical representation of models. To make a model form apply to a specific situation, we must estimate or guess what these values are; in this way we infuse life into the abstract model. Parameters often have direct marketing interpretations (e.g., market potential or price elasticity).

Calibration is the process of determining appropriate values of the parameters. You might use statistical methods (i.e., estimation), some sort of judgmental process, or a combination of approaches.

For example, a simple model is

$$Y = a + bX \,, \tag{1}$$

In Eq. (1), X is an *independent variable* (advertising, say), Y is a *dependent variable* (sales), the model form is linear, and a and b are *parameters*. Note that a in Eq. (1) is the level of sales (Y) when X equals 0 (*zero advertising*), or the *base* sales level. For every dollar increase in advertising, Eq. (1) says that we should expect to see a change in sales of b units. Here b is the slope of the sales/advertising response model. When we determine that the right value of a and b (e.g., via judgmental calibration, or via statistical estimation) are 23,000 and 4, respectively, and place those values in Eq. (1) to get

$$Y = 23{,}000 + 4X \,, \tag{2}$$

then we say we have *calibrated* the model (given values to its parameters) (Exhibit 1).

Some Simple Market Response Models

In this section, we will provide a foundation of simple but widely used models of market response that relate one dependent variable to one independent variable in the absence of competition. The linear model shown in Exhibit 1 is used frequently, but it is far from consistent with the ways markets appear to behave.

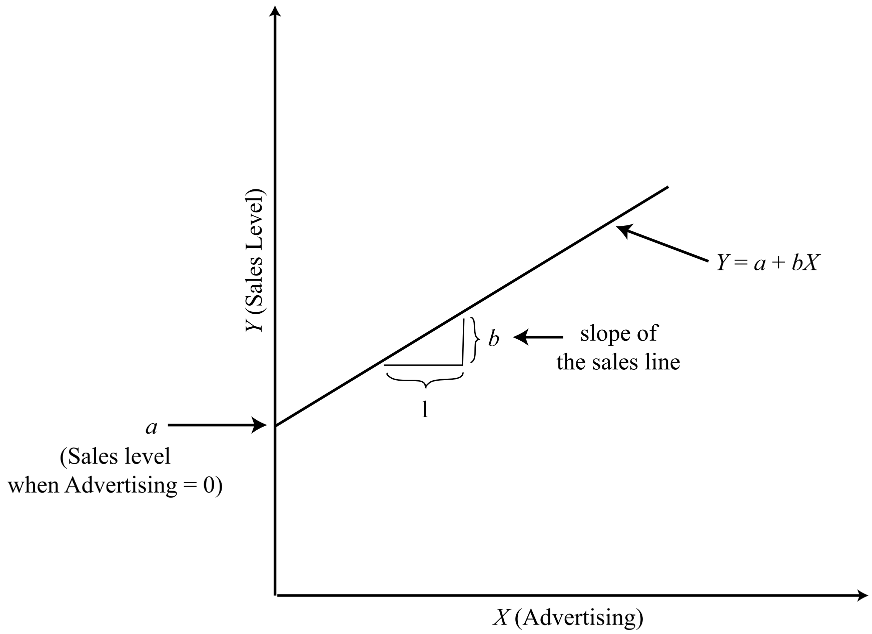

EXHIBIT 1

Interpreting the coefficients of a linear response model.

Saunders (1987) summarizes several phenomena that have been reported in marketing studies and that we should be able to handle using our toolkit of models (Exhibit 2). In describing these eight phenomena here, we use the term *input* to refer to the level of marketing effort (the X or independent variable) and *output* to refer to the result (the Y or dependent variable):

P1. Output is zero when input is zero.

P2. The relationship between input and output is linear.

P3. Returns decrease as the scale of input increases (every additional unit of input gives *less* output than the previous unit gave).

P4. Output cannot exceed some level (saturation).

P5. Returns increase as scale of input increases (every additional unit of input gives *more* output than the previous unit).

P6. Returns first increase and then decrease as input increases (S-shaped return).

P7. Input must exceed some level before it produces any output (threshold).

P8. Beyond some level of input, output declines (supersaturation point).

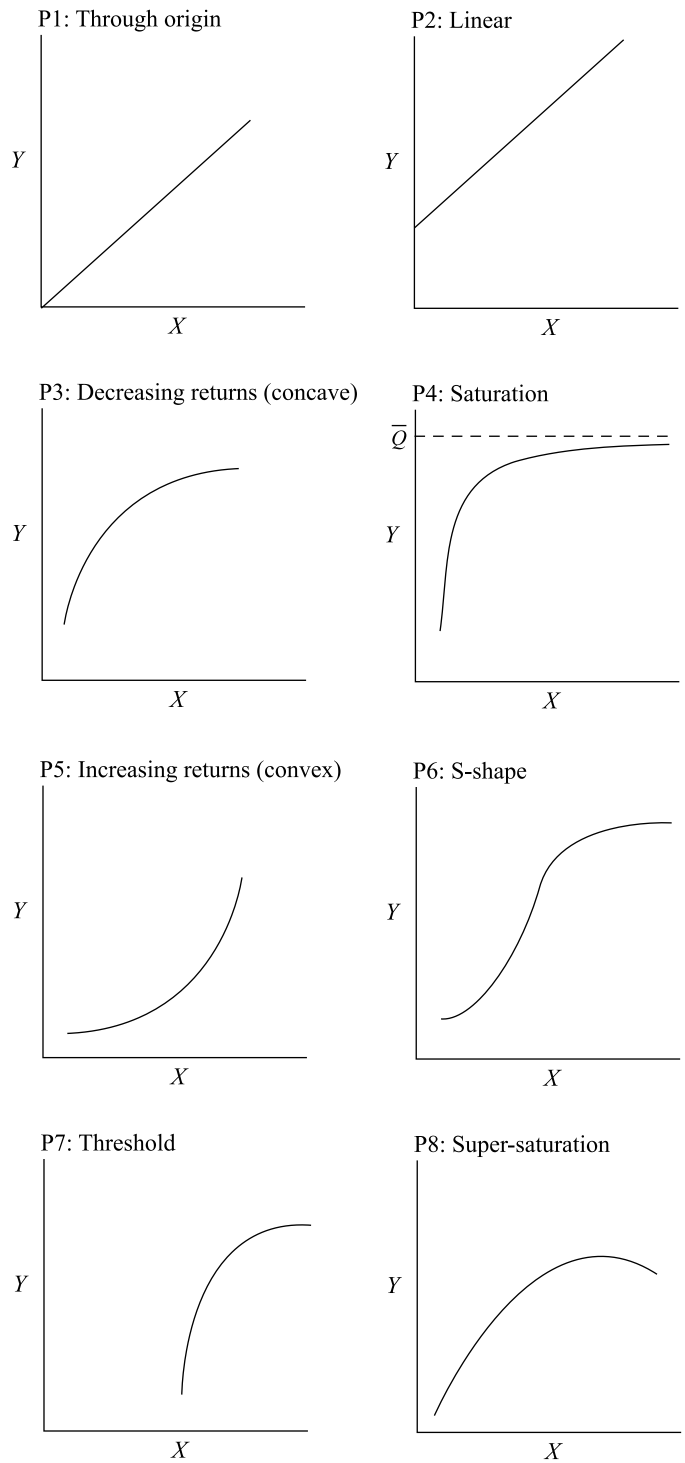

EXHIBIT 2

Pictorial representation of Saunders' response model phenomena.

The phenomena we wish to incorporate in our model of the marketplace depend on many things, including what we have observed about the market (data), what we know about the market (judgment or experience), and existing theory about how markets react. We now outline some of the common model forms that incorporate these phenomena.

The linear model: The simplest and most widely used model is the linear model:

$$Y = a + bX \, , \tag{3}$$

The linear model has several appealing characteristics:

- Given market data, one can use standard regression methods to estimate the parameters.
- The model is easy to visualize and understand.
- Within specific ranges of inputs, the model can approximate many more complicated functions quite well—a straight line can come fairly close to approximating most curves in a limited region.

It has the following problems:

- It assumes constant returns to scale everywhere, i.e., it cannot accommodate P3, P5, or P6.
- It has no upper bound on Y.
- It often gives managers unreasonable guidance on decisions.

On this last point, note that the sales slope (□Y/□X) is constant everywhere and equal to b. Thus if the contribution margin (assumed to be constant, for the moment) is m for the product, then the marginal profit from an additional unit of spending is bm. If $bm>1$, more should be spent on that marketing activity, without limit—that is, every dollar spent immediately generates more than a dollar in profit! If $bm<1$, nothing should be spent. Clearly this model is of limited use for global decision making (It says: spend limitless amounts or nothing at all!), but locally the model suggests whether a spending increase or decrease is appropriate.

Linear models have seen wide use in marketing, and they readily handle phenomena P1 and P2. If X is constrained to lie within a range $\underline{B} \leq X \leq \overline{B}$, the model can accommodate P4 and P7 as well.

The power series model: If we are uncertain what the relationship is between X and Y, we can use a power series model. Here the response model is

$$Y = a + bX + cX^2 + dX^3 + \ldots \qquad (4)$$

which can take many shapes.

The power series model may fit well within the range of the data but will normally behave badly (becoming unbounded) outside the data range. By selecting parameter values appropriately the model may be designed to handle phenomena P1, P2, P3, P5, P6, and P8.

The fractional root model: The fractional root model,

$$Y = a + bX^c \text{ (with c prespecified)} \qquad (5)$$

has a simple but flexible form. There are combinations of parameters that give increasing, decreasing, and (with c=1) constant returns to scale. When c=1/2 the model is called the *square root model*. When c=–1 it is called the *reciprocal model*; here Y approaches the value a when X gets large. If a=0, the parameter c has the economic interpretation of elasticity (the percent change in sales, Y, when there is a 1 percent change in marketing effort X). When X is price, c is normally negative, whereas it is positive for most other marketing variables. This model handles P1, P2, P3, P4, and P5, depending on what parameter values you select.

The semilog model: With the functional form

$$Y = a + b \ln X, \qquad (6)$$

the semilog model handles situations in which constant percentage increases in marketing effort result in constant absolute increases in sales. It handles P3 and P7 and can be used to represent a response to advertising spending where after some threshold of awareness, additional spending may have diminishing returns.

The exponential model: The exponential model,

$$Y = ae^{bX} \text{ where } X > 0, \qquad (7)$$

characterizes situations where there are increasing returns to scale (for b>0); however, it is most widely used as a price-response function for b<0 (i.e., increasing returns to decreases in price) when Y approaches 0 as X becomes large. It handles phenomena P5 and, if b is negative, P4 (Y approaches 0, a lower bound here).

The modified exponential model: The modified exponential model has the following form:

$$Y = a(1 - e^{-bX}) + c, \tag{8}$$

It has an upper bound or saturation level at $a+c$ and a lower bound of c, and it shows decreasing returns to scale. The model handles phenomena P3 and P4 and is used as a response function to selling effort; it can accommodate P1 when c=0.

The logistic model: Of the S-shaped models used in marketing, the logistic model is perhaps the most common. It has the form

$$Y = \frac{a}{1 + e^{-(b+cX)}} + d, \tag{9}$$

This model has a saturation level at $a+d$ and has a region of increasing returns followed by decreasing return to scale; it is symmetrical around $d+a/2$. It handles phenomena P4 and P6, is easy to estimate, and is widely used.

The Gompertz model: A less widely used S-shaped function is the following Gompertz model:

$$Y = ab^{cX} + d, a > 0, 1 > b > 0, c < 1, \tag{10}$$

Both the Gompertz and logistic curves lie between a lower bound and an upper bound; the Gompertz curve involves a constant ratio of successive first differences of log Y, whereas the logistic curve involves a constant ratio of successive first differences of $1/Y$. This model handles phenomena P1, P4, and P6. (The better known logistic function is used more often than the Gompertz because it is easy to estimate.)

The ADBUDG Model: The ADBUDG model, popularized by Little (1970), has the form

$$Y = b + (a - b)\frac{X^c}{d + X^c}, \tag{11}$$

The model is S-shaped for c>1 and concave for 0<c<1. It is bounded between b (lower bound) and a (upper bound). The model handles phenomena P1, P3, P4, and P6, and it is used widely to model response to advertising and selling effort.

Even readers with good mathematical backgrounds may not be able to appreciate the uses, limitations, and flexibility of these model forms. One way to visualize these models would be insert various values of x in Excel, compute the y value corresponding to each x using the formula for the response model, and plot the resulting set of values of y.

Response Model Calibration

Calibration means assigning good values to the parameters of the model. Consider the simple linear model (Eq. 3). If we want to use that model, we have to assign values to a and b. We would want those values to be good ones. But what do we mean by *good*? A vast statistical and econometric literature addresses this question, but we will try to address it simply and intuitively:

Calibration goal: We want estimates of a and b that make the relationship $Y=a+bX$ a good approximation of how Y varies with values of X, which we know something about from data or intuition.

People often use least squares regression to calibrate a model. In effect, if we have a number of observations of X (call them x_1, x_2, etc.) and associated observations of Y (called y_1, y_2, etc.), regression estimates of a and b are those values that minimize the sum of the squared differences between each of the observed Y values and the associated "estimate" provided by the model. For example, $a+bx_7$ would be our estimate of y_7, and we would want y_7 and $a+bx_7$ to be close to each another. We may have actual data about these pairs of X's and Y's or we may use our best judgment to generate them ("What level of sales would we get if our advertising were 10 times what it is now? What if it were half of what it is now?").

When the data that we use for calibration are actual experimental or market data, we call the calibration task "objective calibration" (or objective parameter estimation). When the data are subjective judgments, we call the task "subjective calibration."

In either case we need an idea of how well the model represents the data. One frequently used index is R^2, or R-square. If each of the estimated values of Y equals the actual value of Y, then R-square has a maximum value of 1; if the estimates of Y do only as well as the average of the Y values, then R-square has a value of 0. If R-square is less than 0, then we are doing worse than we would by simply assigning the average value of Y to every value of X. In that case we have a very poor model indeed!

Formally R-square is defined as

$$R^2 = 1 - \frac{\text{(Sum of squared differences between actual Y's and estimated Y's)}}{\text{(Sum of squared differences between actual Y's and the average value of Y)}} \quad (12)$$

EXAMPLE

Suppose we have run an advertising experiment across a number of regions with the following results:

Region	Annual Advertising (per capita)	Annual Sales Units (per capita)
A	\$ 0	5
B	2	7
C	4	13
D	6	22
E	8	25
F	10	27
G	12	31
H	14	33

Let us take the ADBUDG function (Eq. 2.11). If we try to estimate the parameters of the ADBUDG function (a, b, c, d) for these data, to maximize the R-square criterion, we get

$$\hat{a} = 39.7, \hat{b} = 4.6, \hat{c} = 2.0, \hat{d} = 43.4, \text{ with } R^2 = 0.99, \quad (13)$$

In Exhibit 3 we plot the results. The plot shows how well the response model fits these data.

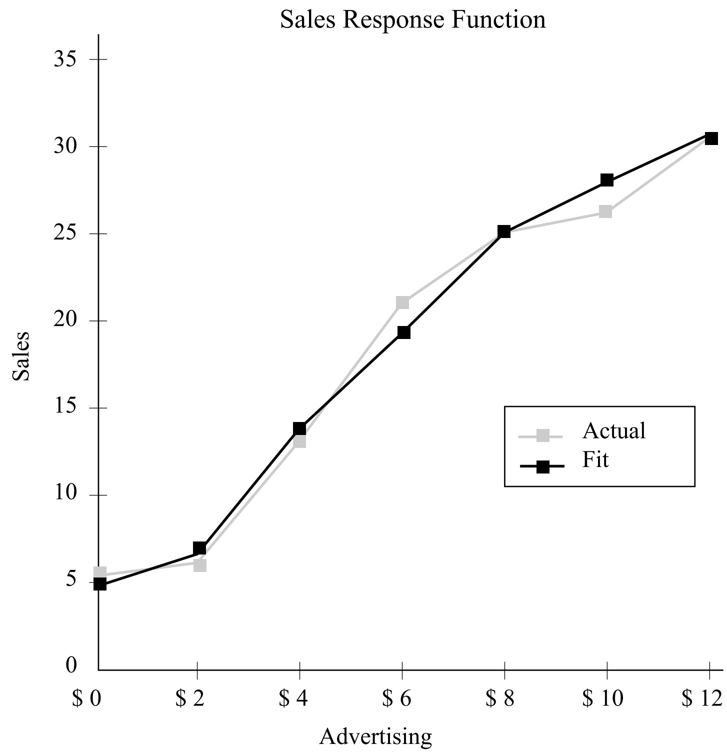

EXHIBIT 3

Calibration example using ADBUDG function, with R^2=0.99.

In many cases managers do not have historical data that are relevant for calibrating the model for one of several reasons. If the firm always spends about the same amount for advertising (say 4 percent of sales in all market areas), then it has no objective information about what would happen if it changed the advertising-to-sales ratio to 8 percent. Alternatively, the firm may have some historical data, but that data may not be relevant because of changes in the marketplace such as new competitive entries, changes in brand-price structures, changes in customer preferences, and the like. (Consider the problem of using year-old data in the personal computer market to predict future market behavior.)

To formally incorporate managerial judgment in a response function format, Little (1970) developed a procedure called "decision calculus." In essence, decision calculus asks the manager to run a mental version of the previous market experiment.

Q1: What is our current level of advertising and sales?
Ans.: Advertising=$8/capita; sales=25 units/capita.

Q2: What would sales be if we spent $0 in advertising? (*A*=$0/capita)

Q3: What would sales be if we cut 50 percent from our current advertising budget?
(*A*=$4/capita)

Q4: What would sales be if we increased our advertising budget by 50 percent? (*A*=$12/capita)

Q5: What would sales be if advertising were made arbitrarily large?
(*A*=$•/capita)

Suppose that the manager answered Questions 2 through 5 by 5, 13, 31, and 40, respectively; we would get essentially the same sales response function as in the previous example.

Multiple Marketing-Mix Elements: Interactions

In the previous section we dealt with market response models of one variable. When we consider multiple marketing-mix variables, we should account for their interactions. As Saunders (1987) points out, interactions are usually treated in one of three ways: (1) by assuming they do not exist, (2) by assuming that they are multiplicative, or (3) by assuming they are multiplicative and additive. For example, if we have two marketing-mix variables X_1 and X_2 with individual response functions $f(X_1)$ and $g(X_2)$, then assumption (1) gives us

$$Y = af(X_1) + bg(X_2); \tag{14}$$

assumption (2) gives us

$$Y = af(X_1)g(X_2); \tag{15}$$

and assumption (3) gives us

$$Y = af(X_1) + bg(X_2) + cf(X_1)g(X_2), \tag{16}$$

In practice when multiple marketing-mix elements are involved, we can resort to one of two forms: the (full) linear interactive form or the multiplicative form. The full linear interactive model (for two variables) takes the following form:

$$Y = a + bX_1 + cX_2 + dX_1X_2, \tag{17}$$

Note here that $\partial Y/\partial X_1 = b + dX_2$, so that sales response to changes in marketing-mix element X_1 is affected by the level of the second variable, X_2.

The multiplicative form is as follows:

$$Y = aX_1^b X_2^c, \tag{18}$$

Here $\partial Y/\partial X_1 = abX_1^{b-1} X_2^c$, so that the change in the response at any point is a function of the levels of both independent variables. Note here that b and c are the *constant* elasticities of the first and second marketing-mix variables, respectively, at all effort levels X_1 and X_2.

Dynamic Effects

Response to marketing actions does not often take place instantly. *Carryover effects* is the general term used to describe the influence of a current marketing expenditure on sales in future periods (Exhibit 4). We can distinguish several types of carryover effects. One type, the *delayed-response effect*, arises from delays between when marketing dollars are spent and their impact. Delayed response is especially evident in industrial markets, where the delay, especially for capital equipment, can be a year or more. Another type of effect, the *customer-holdover effect*, arises when new customers created by the marketing expenditures remain customers for many subsequent periods. Their later purchases should be credited to some extent to the earlier marketing expenditures. Some percentage of such new customers will be retained in each subsequent period; this phenomenon gives rise to the notion of the *customer retention rate* and its converse, the *customer decay rate* (also called the attrition or erosion rate).

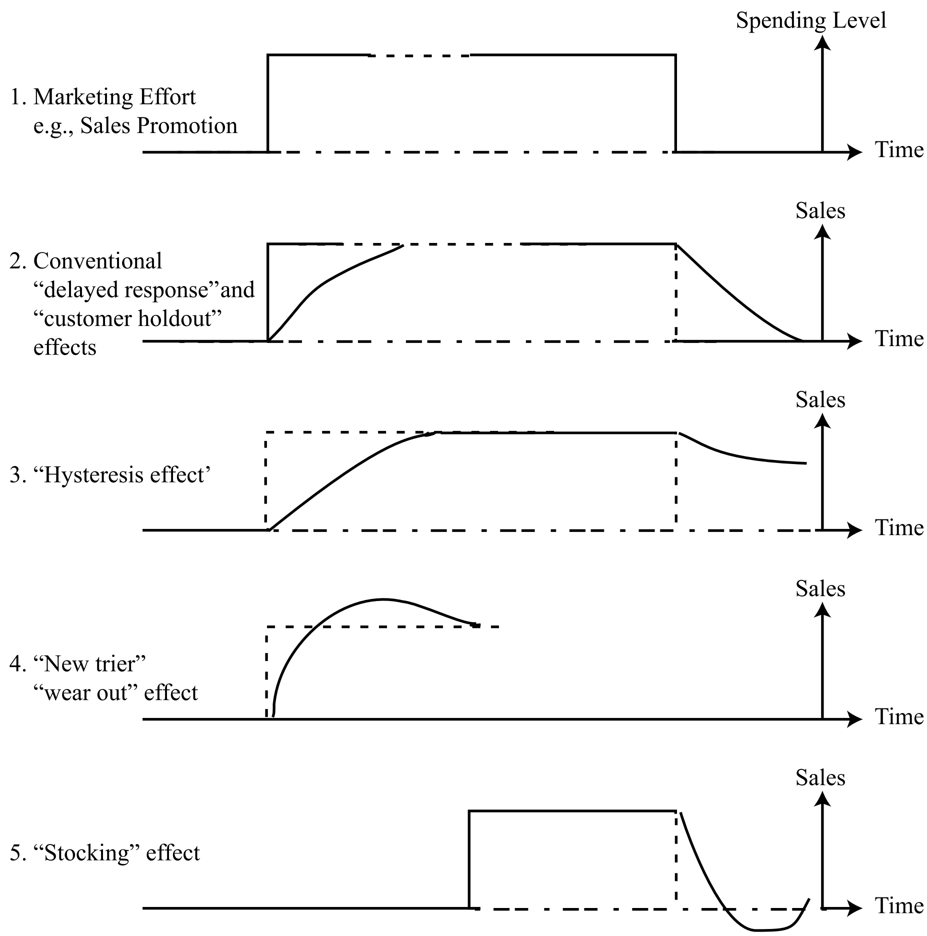

EXHIBIT 4

Some types of dynamic marketing responses. Source: Saunders 1987, p. 33.

A third form of delayed response is *hysteresis*, the asymmetry in sales buildup compared with sales decline. For example, sales may rise quickly when an advertising program begins and then remain the same or decline slowly after the program ends.

New trier effects, in which sales reach a peak before settling down to steady state, are common for frequently purchased products, for which many customers try a new brand but only a few become regular users.

Stocking effects occur when a sales promotion not only attracts new customers but encourages existing customers to stock up or buy ahead. The stocking effect often leads to a sales trough in the period following the promotion (4).

The most common dynamic or carryover effect model used in marketing is

$$Y_t = a_0 + a_1 X_t + \lambda Y_{t-1}, \tag{19}$$

Eq. (19) says that sales at time t (Y_t) are made up of a constant minimum base (a_0), an effect of current activity $a_1 X_t$, and a proportion of last period's sales (l) that carries over to this period. Note that Y_t is influenced to some extent by all previous effort levels X_{t-1}, X_{t-2}, ..., X_0, because Y_{t-1} depends on X_{t-1} and Y_{t-2}, and in turn Y_{t-2} depends on X_{t-2} and Y_{t-3}, and so on. The simple form of Eq. (19) makes calibration easy—managers can either guess l directly as the proportion of sales that carries over from one period to the next or estimate it by using linear regression.

Market-Share Models and Competitive Effects

Thus far we have ignored the effect of competition in our models, assuming that product sales result directly from marketing activities. Yet, if the set of product choices characterizing a market are well defined, we can specify three types of models that might be appropriate:

- Brand sales models (Y)
- Product class sales models (V)
- Market-share models (M)

Note that by definition

$$Y = M \times V, \tag{20}$$

Models of product class sales (V) have generally used many of the analytic forms we have introduced earlier, explaining demand through environmental variables (population sizes, growth, past sales levels, etc.) and by aggregate values of marketing variables (total advertising spending, average price, etc.). Market-share models have a different structure. To be logically consistent, regardless of what any competitor does in the marketplace, each firm's market share must be between 0 and 100 percent (range restriction) and market shares, summed over brands, must equal 100 percent (sum restriction). A class of models that satisfy both the range and the sum restrictions are attraction models, where the attraction of a brand depends on its marketing mix. Essentially these models say our share=us/(us+them), where "us" refers to the attractiveness of our brand and (us+them) refers to the attractiveness of all brands in the market, including our brand.

Thus the general attraction model can be written as

$$M_i = \frac{A_i}{A_1 + A_2 + \ldots + A_n}, \tag{21}$$

where

A_i = attractiveness of brand i, and with at least one and

M_i = firm i's market share.

Attraction models suggest that the market share of a brand is equal to the brand's share of the total marketing effort (attractiveness).

While many model forms of A's are used in practice, two of the most common are the linear interactive form and the multiplicative form outlined in the section on interactions of marketing-mix elements. Both of these models suffer from what is called the "proportional draw" property. We can see this best via an example:

EXAMPLE

Suppose A_1=10, A_2=5, and A_3=5.

In a market with A_1 and A_2 only,

$$m_1 = \frac{10}{10+5} = 66\tfrac{2}{3}\% \text{ and } m_2 = \frac{5}{10+5} = 33\tfrac{1}{3}\%,$$

Suppose A_3 enters. Then after entry,

$$\overline{m_1} = \frac{10}{10+5+5} = 50\%, \overline{m_2} = 25\%, \text{ and } \overline{m_3} = 25\%,$$

Note that brand 3 draws its 25 percent market share from the other two brands, $16^2/_3$ percent from brand 1 and $8^1/_3$ percent from brand 2—that is, proportional to those brands' market shares. But suppose that brand 3 is a product aimed at attacking brand 1; one would expect it to compete more than proportionally with brand 1 and less than proportionally with brand 2.

Thus when using simple market-share models, ensure that all the brands you are considering are competing for essentially the same market. Otherwise you will need to use extensions of these basic models that admit different levels of competition between brands (Cooper 1993).

We did not include here a description of individual-level response models. This is an important type of response model that is gaining wide use within marketing, especially in direct marketing and CRM applications. In a separate technical note titled Choice Modeling, we provide a detailed description of a widely used individual-level response model called Multinomial Logit.

Summary

We summarize several commonly used aggregate market response models, and describe their properties. We show how complexity increases even for modeling aggregate market response when we introduce realistic aspects of marketing phenomena, such as interactions among marketing variables, dynamic response over time, or, competitive effects. We also briefly describe how aggregate marketing response functions could be estimated either via judgmental calibration or statistical estimation.

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